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# SHORTER COMMUNICATIONS

## ON HEAT TRANSFER WITH MOVING BOUNDARY

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### NOMENCLATURE

$a$, thermal diffusivity [$m^2/s$];
$A$, object surface area [$m^2$];
$Bi$, Biot number $= \alpha D/\lambda$;
$C_p$, polymer heat capacity [J/kg°C];
$C_v$, object constant, $\rho C_p V/A$ [J/m$^2$°C];
$D$, coating thickness [m];
$\Delta H$, heat of fusion [J/kg];
$p'$, any property of polymer particles;
$t$, time [s];
$T$, temperature [°C];
$x$, co-ordinate;
$V$, object volume [$m^3$];
$\alpha$, heat-transfer coefficient [W/m$^2$°C];
$\eta$, dummy variable;
$\theta_m$, dimensionless temperature, $= (T_m - T_b)/(T_{v0} - T_b)$;
$\lambda$, thermal conductivity of polymer [W/m°C];
$\rho$, density [kg/$m^3$].

Indices

$b$, bed;
$m$, melting;
$0$, start values;
$v$, object;
$\infty$, maximum;
$\langle\,\rangle$, average;
$*$, transformed co-ordinates.

### INTRODUCTION

THE PROBLEM of heat transfer with moving boundary is also encountered in fluidized bed dip coating process. The process consists of fluidizing the polymer powder and then dipping the heated object into the bed. Upon withdrawal of the object, after a desired residence time in the bed, a molten plastic layer is observed to have formed on the object surface which later congeals into a more or less continuous protective film.

In the present study a general theory of fluidized bed coating has been developed and compared with other studies reported in the literature.

### THE SOLUTION OF THE PROBLEM

Two books [1, 2] and several papers [3, 4] which have recently appeared independently on the theory of fluidized bed coating deserve close attention. In one of the papers [4] an attempt has been made to solve the heat transfer with moving boundary problem which is encountered in fluidized bed coating processes. Some of the restrictions, i.e. the constant object temperature, the constant polymer properties, imposed in solving the problem in our view are not necessary. For the solution of the problem has already been given in an internal report as early as at the beginning of 1968 [2]. Below we give the highlights of this theoretical work.

The set of equations describing the process are:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(a\frac{\partial T}{\partial x}\right), \qquad 0 \leqslant x \leqslant D(t), \tag{1}$$

$$x = 0: T(0, t) = T_v(t); T(0, 0) = T_{v0}, \tag{2}$$

$$x = D(t): \lambda\frac{\partial T}{\partial x} = \alpha(T_m - T_b) + \{\rho C_p(T_m - T_b) + \rho\Delta H\}\frac{dD}{dt}, \tag{3}$$

$$T[D(t), t] = T_m, \tag{4}$$

$$T(x, 0) = T_b, \tag{5}$$

$$D(0) = 0,$$

where $D(t)$ is the time dependent coating thickness. $T_b$, $T_{v0}$, $T_v$, $T_m$ are the bed, the initial, object, the instantaneous object and the melting temperatures of the polymer respectively. In general any polymer property can be averaged as [2]:

$$\langle p'\rangle = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} p'(T')\,dT'. \tag{6}$$

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The solution of the above set of equations can be accomplished by making the following transformations:

$$x^* = x/D(t);\quad t^* = at;\quad T^* = T/T_m.$$

Under these transformations the above equations become

$$D^2\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{x^*}{2}\cdot\frac{\mathrm{d}D^2}{\mathrm{d}t^*}\cdot\frac{\partial T^*}{\partial x^*} \tag{7}$$

$$x^* = 0: T^*(0, t^*) = T_r^*: T^*(0, 0) = T_{r0}^*, \tag{8}$$

$$x^* = 1: \left.\frac{\partial T^*}{\partial x^*}\right|_{x^*=1} = -\left\{(1 - T_b^*)Bi + \frac{1}{2}\left[(1 - T_b^*) + \frac{\Delta H}{C_pT_m}\right]\frac{\mathrm{d}D^2}{\mathrm{d}t^*}\right\}, \tag{9}$$

$$T^*(1, t^*) = 1, \tag{10}$$

$$T^*(x^*, 0) = T_b^*, \tag{11}$$

$$D(0) = 0, \tag{12}$$

where $Bi (= \alpha D/\lambda)$ is known as Biot number.

Integration of the new equation (7) from $x^* = 1$ to $x^*$ gives

$$D^2\frac{\partial}{\partial t^*}\int_1^{x^*} T^*(\eta, t^*)\,\mathrm{d}\eta = \frac{\partial T^*}{\partial x^*} + \Delta + \frac{1}{2}\frac{\mathrm{d}D^2}{\mathrm{d}t^*} \times \left\{x^*T^* - 1 - \int_1^{x^*} T^*\,\mathrm{d}\eta\right\}, \tag{13}$$

where

$$\Delta = (1 - T_b^*)Bi + \frac{1}{2}\left[(1 - T_b^*) + \frac{\Delta H}{C_pT_m}\right]\frac{\mathrm{d}D^2}{\mathrm{d}t^*}.$$

A second integration and the rearrangement of the last equation and after the insertion of the boundary condition at $x^* = 0$ produces

$$\frac{\mathrm{d}}{\mathrm{d}t^*}\left[D^2\left\{\int_0^1 \eta T^*\,\mathrm{d}\eta + \frac{1}{2}\left[(1 - T_b^*) + \frac{\Delta H}{C_pT_m}\right]\right\}\right] = T_r^* - 1 - (1 - T_b^*)Bi. \tag{14}$$

The last equation can be integrated with respect to time

$$D^2 = \frac{\int_0^{t^*}\{T_r^* - 1 - (1 - T_b^*)Bi\}\,\mathrm{d}t^{*\prime}}{\int_0^1 \eta T^*\,\mathrm{d}\eta + \frac{1}{2}\left(\frac{\Delta H}{C_pT_m} - T_b^*\right)}. \tag{15}$$

Equation (15) gives $D$, the coating thickness, in terms of the temperature profile in the coating and the boundary conditions at the object surface. Since it involves integrals it has the usual advantage of integral formulation in that it is relatively insensitive to small variations in the kernel function in the integrals.

Instead of assuming a second degree polynomial or a higher one for the temperature profile in the coating layer as has been done in [4] we shall find the upper and the lower bounds on the integral of equation (15). Such a procedure has the advantage that the final solution for $D(t)$ is not very sensitive to the form of the temperature profile in the coating layer.

For moderate coating conditions such that

$$(T_{r0}^* - 1)\bigg/\left(\frac{\Delta H}{C_pT_m} - T_b^* + 1\right) \ll 1,$$

and since that for the bounds $1 \leqslant T^* \leqslant T_r^*$, [2], then equation (15) reduces to

$$D^2 = \frac{2\int_0^{t^*}\{T_r^*(t^{*\prime}) - 1 - (1 - T_b^*)Bi\}\mathrm{d}t^{*\prime}}{\frac{\Delta H}{C_pT_m} - T_b^* + \frac{1}{2}(T_{r0}^* + 1)}, \tag{16}$$

where $T_r^*$ is given by a simple heat balance taken at the coating–object interface [2]:

$$(T_r^* - 1)/(T_{r0}^* - 1) = \exp\{-(2\lambda/C_r)(t^*/\pi a^2)^{\frac{1}{2}}\},$$

with $C_r$ being the object constant.

Equation (16) is the most general solution giving the coating. Unlike equation

$$t = \int_0^D \left\{\frac{\frac{1}{3a}\left[2\theta + \frac{\alpha}{\lambda}\eta + 5 + \frac{1}{2}F(\eta) + \frac{(\alpha/\lambda)\eta[(\alpha/\lambda)\eta - 2]}{2F(\eta)}\right]}{4\theta + 2 - 3\frac{\alpha}{\lambda}\eta - F(\eta)}\right\}$$

with

$$F(\eta) = \left\{\left(\frac{\alpha}{\lambda}D - 2\right)^2 + 8\theta\right\}$$

and

$$\theta = (T_r - T_m)/(T_m - T_b),$$

given in [4], the above equation is structurally simple and the values of $D$ can be easily calculated as function of known parameters. However the last equation can also be further simplified by considering short coating times, i.e. when $T_r^* \to T_{r0}^*$. In which case equation (16) reduces to

$$t = \frac{D^2}{4a}\cdot\frac{T_{r0}^* - 2T_b^* + 2(\Delta H/C_pT_m) + 1}{T_{r0}^* - 1 - (1 - T_b^*)\langle Bi\rangle}, \tag{17}$$

where $\langle Bi\rangle$ is the average value of between $t = 0$ and $t$.

Some useful relationships may be derived from equation (17). Letting $t \to \infty$ we obtain

$$\langle Bi_\infty\rangle = \frac{T_{r0}^* - 1}{1 - T_b^*}. \tag{18}$$

and

$$\langle D_x \rangle = \frac{\lambda}{\alpha} \cdot \frac{1 - \theta_m}{\theta_m}. \tag{19}$$

where $\theta_m$ is the dimensionless temperature.

In case of some polymers where $\Delta H / C_p T_m < 1$ and in dense fluidized beds where $\alpha/\lambda \to 0$ equation (17) reduces to

$$\frac{D}{\sqrt{at}} = 2\{(1 - \theta_m)/(1 + \theta_m - \theta_m^2)^{\frac{1}{2}}\}$$

$$\text{or } \frac{D}{\sqrt{at}} \simeq 2(1 - \theta_m), \qquad \text{for } \theta_m < 0.5, \tag{20}$$

which again is more simple than the following equation given in [4];

$$\frac{D}{\sqrt{at}} = \left\{\frac{12[2\theta + 1 - \sqrt{(2\theta + 1)}]}{2\theta + 5 + \sqrt{(2\theta + 1)}}\right\}^{\frac{1}{2}}.$$

The extensive experimental results given elsewhere [1, 2, 5] verify all of the above theoretical predictions better than with 10 per cent accuracy.

## CONCLUSIONS

In this paper an attempt has been made to give a general theory for the process of dip-coating in a fluidized bed. Although the problem is that of finding the solution of a heat transfer problem with moving boundary, unfortunately the presence of the moving boundary makes the equations non-linear, thus preventing one obtaining an analytical solution. However, by integrating the set of equations describing the process it has been possible to find the upper and the lower bounds of the coating thickness as function of time.

The theoretical results have been compared with their literature counterparts and they are found to be more general and more suitable for numerical calculations than the results reported in [4].

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# EDDY-SHEDDING FROM A SPHERE IN TURBULENT FREE-STREAMS

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## NOMENCLATURE

- $D$, sphere diameter [cm];
- $L$, integral scale of turbulence [cm];
- $n$, frequency of eddy shedding [Hz];
- $U$, free-stream velocity [cm/s].

Greek symbol

- $\nu$, kinematic viscosity [cm²/s].

Dimensionless numbers

- $Re$, Reynolds number, $DU/\nu$;
- $St$, Strouhal number, $nD/U$.

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THE PRESENT note deals with the recent paper by Raithby and Eckert [1] in which they have reported the effect of turbulence intensity, the scale of turbulence, and the position of the support on macroscopic heat transfer from spheres to an air stream in the Reynolds number range $3.6 \times 10^3$–$5.2 \times 10^4$, the Reynolds number being based on the sphere diameter, $D$, and the free-stream velocity, $U$.